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Using Rolling-Window Multilateral Price Indexes to Track Food Costs Across Space and Over Time

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Why do we need price indexes?

- Inflation surveillance:
  - price indexes compare costs of *identical or similar* products.
  - unit values (e.g., $ per pound of fruit) could be literally comparing (the cost of) apples to oranges.
- Demand estimation:
  - consistency requires the price variables be driven by exogenous market conditions.
  - price indexes control for between-product cost differentials.
  - unit values register (endogenous) choice over quality—famous unit value bias (Cox and Wohlgenant 1986; Deaton 1988).

Common price indexes

- Bilateral price indexes:
  - Laspeyres (1871), Paasche (1874), Fisher Ideal (Fisher 1922), and Törnqvist (1936).
- Multilateral price indexes:
  - GEKS (Gini 1931; Eltetö and Köves 1964; Szulc 1964) and CCD (Caves, Christensen, and Diewert 1982).
- Rolling-window price indexes
  - Rolling-window GEKS (Ivancic, Diewert, and Fox 2011) and CCD (Zhen, Finkelstein et al. 2019) indexes.

Bilateral price indexes

Laspeyres $P_L^{0j} = \frac{\sum_{v \in v_{0j}} p_v^j q_v^0}{\sum_{v \in v_{0j}} p_v^0 q_v^0}$

Key properties: 1) weight barcodes according to their base quantities, 2) hold standard of living (i.e., utility) constant between base and *j*, and 3) not minimum cost at entity *j* prices

Paasche $P_P^{0j} = \frac{\sum_{v \in v_{0j}} p_v^j q_v^j}{\sum_{v \in v_{0j}} p_v^0 q_v^j}$

Key properties: 1) weight barcodes according to their entity *j* quantities, 2) minimum cost at entity *j* prices, and 3) different utility levels at base and *j*

Superlative price indexes

Fisher Ideal $P_F^{0j} = \sqrt{P_L^{0j} \times P_P^{0j}}$

Key properties:

1) account for substitution between barcodes
2) consistent with the (homothetic) "quadratic mean of order two" cost function $c(\mathbf{p}^j) = \left[\sum_k \sum_l b_{kl} p_k^j p_l^j\right]^{1/2}$, where unit cost c is related to the utility level u and total cost C by $c = uC$
3) no reference utility (i.e., whether it is cost of living at u^0 or u^j is irrelevant)
4) homotheticity implies unit expenditure elasticities for all barcodes

Superlative price indexes (continued)

Törnqvist $P_T^{0j} = \exp\left\{0.5 \sum_{v \in v_{0j}} \left(s_v^0 + s_v^j\right) \ln\left(p_v^j / p_v^0\right)\right\}$

Key properties:

1) account for substitution between barcodes
2) consistent with the (homothetic) translog unit cost function $\ln c\left(\mathbf{p}^j\right) = a_0 + \sum_k a_k \ln p_k^j + 0.5 \sum_k \sum_l g_{kl} \ln p_k^j \ln p_l^j$, and the (nonhomothetic) translog total cost function $\ln C\left(u, \mathbf{p}^j\right) = a_0 + \sum_k a_k \ln p_k^j + 0.5 \sum_k \sum_l g_{kl} \ln p_k^j \ln p_l^j + b \ln u + c(\ln u)^2 + \sum_k d_k \ln u \ln p_k^j$

1) when nonhomothetic, reference utility is $\sqrt{u^0 u^j}$
2) theoretically problematic b/c reference utility changes from one entity to the next, but empirically fine b/c the role of reference utility is numerically trivial

Multilateral price indexes

A price index is transitive if the price ratio between entities i and j is the same whether i and j are compared directly or indirectly through a third entity k.

- ❑ Transitivity is especially important for spatial price comparison.
- ❑ The bilateral Laspeyres, Paasche, Fisher Ideal, and Törnqvist are not transitive.
- ❑ One solution is to create a $M \times M$ matrix of bilateral index numbers
- ❑ A better solution is to create a $M \times 1$ column of index numbers using a multilateral index

The GEKS multilateral price index is calculated as

$$P_{GEKS}^{0j} = \prod_{l=0}^{M} \left(P_F^{0l} \times P_F^{lj}\right)^{1/(M+1)}$$

Properties of Multilateral price indexes

- Replacing the Fisher Ideal index in GEKS with the Törnqvist index produces the multilateral CCD index
- Choice of base entity irrelevant (either an artificial base representing national average prices and sales or any market-period)
- Fully transitive
- Violates characteristicity (e.g., I want to compare Atlanta with Boston. Why should prices in New York City matter?)

The rolling-window GEKS and CCD indexes

- ❑ Fixed-basket indexes become less representative over time. Chaining by a moving base is prone to a downward chain drift.
- ❑ Multilateral indexes avoid chain drift but require revision of published index numbers when new data arrive.
- ❑ Solve both by opening a rolling window (RW) to multilateral indexes.

The RWGEKS index for entity k in month $T+1$ ($T =$ end of the base period) is

$$P_{RWGEKS}^{0k} = P_{GEKS}^{0j} \prod_{l \in I_{T+1:T-11}} \left(P_F^{jl} \times P_F^{lk}\right)^{1/M_{T+1:T-11}}$$

P_{GEKS}^{0j} is link entity j in T; $I_{T+1:T-11}$ is the set of all entities between $T-11$ and $T+1$ (window = one year); and $M_{T+1:T-11}$ is the number of entities in the set $I_{T+1:T-11}$.

Properties of RWGEKS and RWCCD indexes

- Replacing P_{GEKS}^{0j} with P_{CCD}^{0j} and P_F with P_T gives the RWCCD index
- Numerically identical to GEKS and CCD for entities in the base period
- Can be free of chain drift (window length ≥ 1 year to be sure).
- Not fully transitive for all entities.
- Transitive between entities of the same period if they share the same link entity j