Sample-based calibration of multiple surveys

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Outline

1. Introduction: recreational fishing surveys
2. Sample-based calibration
3. Application
4. Conclusions
Marine Recreational Information Program (MRIP)

- NOAA Fisheries is responsible for managing marine fisheries under the Magnuson-Stevens Act
  - MRIP produces estimates of marine recreational catch in US waters
  - input into stock assessment models, used to set annual catch limits
- In MRIP, multiple surveys are combined to create estimates
  - Access Point Angler Intercept Survey (APAIS)
  - Fishing Effort Survey (FES)
  - (others)

- APAIS: stratified multi-stage sample of fishing trips, collecting detailed data on trip and catch characteristics

- FES: stratified sample of general population households, collecting data on number of fishing trips over past 2 months
MRIP Estimation

- Combination of APAIS and FES:
  - survey weights in APAIS are calibrated to FES-obtained estimates of number of trips by state and wave
  - other adjustments for undercoverage of respective frames
- NOAA Fisheries provides public-use datasets with trip-level data and calibrated weights
- Variance estimation: current method uses linearization based on APAIS design
  - does not account for calibration to FES
Sample-based Calibration

- Calibration reduces the variance of survey estimators, so it is generally beneficial to account for it in variance estimation
  - In particular, variance of estimated control totals is zero (for population-based controls)
- But: *sample-based* calibration equalizes estimates between surveys, may not reduce variance
  - Important to account for variance contributions from both surveys into final variance estimates
- We describe methods to incorporate calibration into replicate variance estimation, when calibration totals are themselves random
2. Methodology: Primary Survey (APAIS)

- Sample $s$, weights $w_i$
- Population total $t_y = \sum_U y_i$ estimated by $\hat{t}_y = \sum_s w_i y_i$
  - e.g. $\hat{t}_y$ = estimated total catch of striped bass by private boat in GA during May-June 2019
- Replication variance estimator

$$\hat{V}(t_y) = A \sum_{r=1}^{R} \left( \hat{t}_{yr} - \hat{t}_y \right)^2$$

with $\hat{t}_{yr} = \sum_s w_{ir} y_i$

- Replicate weights $w_{ir}, r = 1, \ldots, R$ and constant $A$ determined by replication method
  - Balanced Repeated Replication (BRR) with Fay’s adjustment
  - $R = 160$
Calibration Survey (FES)

- Sample $s_C$, weights $w_{Ci}$
- Estimator $\hat{t}_{Cx} = \sum_{s_C} w_{Ci} x_i$ of length $H$, to be used as controls
  - e.g. $\hat{t}_{Cx,h}$ = estimated number of angler trips by private boat in GA during May-June 2019
- Estimator $\hat{V}_C(\hat{t}_{Cx})$ of $H \times H$ variance-covariance matrix $\text{Var}(\hat{t}_{Cx})$
Calibration of Primary Survey: Regression Estimation

Regression estimator with calibration vector $\hat{t}_{Cx}$

$$\hat{t}_{y,\text{reg}} = \hat{t}_y + (\hat{t}_{Cx} - \hat{t}_x) \beta = \sum_s w^*_i y_i$$

Define $e_i = y_i - \beta^T U x_i$, then asymptotic variance

$$\text{AVar}(\hat{t}_{y,\text{reg}}) = \text{Var}(\hat{e}_i) \quad \text{(variance with fixed controls)}$$

$$+ \beta^T U \text{Var}(\hat{t}_{Cx}) \beta_U \quad \text{(effect of random controls)}$$

For fixed $\hat{t}_{Cx}$, $\text{Var}(\hat{e}_i)$ consistently estimated by

$$\hat{V}(\hat{t}_{y,\text{reg}}) = A \sum_{r=1}^{R} \left( \hat{t}_{y,\text{reg}}^{(r)} - \hat{t}_{y,\text{reg}} \right)^2$$

with

$$\hat{t}_{y,\text{reg}}^{(r)} = \hat{t}_y^{(r)} + (\hat{t}_{Cx}^{(r)} - \hat{t}_x^{(r)}) \beta^{(r)} = \sum_s w^*_{i}^{(r)} y_i$$

(“apply calibration to each replicate”)
Approaches to Estimate $\text{AVar}(\hat{t}_{y,\text{reg}})$

1. Direct plug-in: $\hat{V}(\hat{t}_e) + \hat{\beta}^T \hat{V}(\hat{t}_{Cx})\hat{\beta}$

2. Opsomer and Erciulescu (2021): when replicates are available for both surveys, create replicated control totals $\hat{t}_{Cx}^{(r)}$ to calibrate primary survey replicates (originally proposed by Kott (2005))

3. Fuller (1998): compute \textit{eigen-decomposition} of $\hat{V}(\hat{t}_{Cx})$ and perturb controls of primary survey replicates
Implementing Opsomer and Erciulescu (2021) Method

- Applicable when control survey has replicates for variance estimation
- Estimate vector $\hat{t}_{Cx} = \sum_{sC} w_{Ci} x_i$
- Replicate variance-covariance matrix estimator

$$\hat{V}_C(\hat{t}_{Cx}) = A_C \sum_{r=1}^{R_C} \left( \hat{t}_{Cx}^{(r)} - \hat{t}_{Cx} \right) \left( \hat{t}_{Cx}^{(r)} - \hat{t}_{Cx} \right)^T$$

with $\hat{t}_{Cx}^{(r)} = \sum_{sC} w_{Ci}^{(r)} x_i$

- Replicate weights $w_{Ci}^{(r)}, r = 1, \ldots, R_C$ and constant $A_C$ determined by control survey replication method
- Assume $R_C = R$
Implementing Opsomer and Erciulescu (2021) Method (2)

- Adjust control totals in replicates of primary survey, based on replicates from control survey

$$
\hat{t}_{y,\text{reg}}^{(r)} = \hat{t}_y^{(r)} + (\hat{t}_{Cx} + a_r(\hat{t}_{Cx} - \hat{t}_x) - \hat{t}_x^{(r)})^T\hat{\beta}^{(r)}
$$

$$
= \hat{t}_y^{(r)} + (t_{Cx} - \hat{t}_x^{(r)})^T\hat{\beta}^{(r)} + a_r(t_{Cx} - \hat{t}_C)^T\hat{\beta}^{(r)}
$$

- Set $a_r = \sqrt{A_C/A}$, then

$$
\hat{V}(\hat{t}_{y,\text{reg}}) = A \sum_{r=1}^{R} \left( \hat{t}_{y,\text{reg}}^{(r)} - \hat{t}_{y,\text{reg}} \right)^2
$$

consistent for

$$
A\text{Var}(\hat{t}_{y,\text{reg}}) = \text{Var}(\hat{t}_e) + \beta_U^T\text{Var}(\hat{t}_{Cx})\beta_U
$$
Implementing Fuller (1998) Method

- Assume $H = R$ for now
- Compute eigen-decomposition of $\hat{V}(\hat{t}_{Cx})$

$$\hat{V}(\hat{t}_{Cx}) = \sum_{h=1}^{H} \lambda_h q_h q_h^T = \sum_{h=1}^{H} \delta_h \delta_h^T$$

- Adjust control totals in replicates of primary survey

$$\hat{t}_{y,\text{reg}}^{(r)} = \hat{t}_y^{(r)} + (\hat{t}_{Cx} + a_r \delta_r - \hat{t}_x^{(r)}) \hat{\beta}^{(r)}$$

$$= \hat{t}_y^{(r)} + (\hat{t}_{Cx} - \hat{t}_x^{(r)}) \hat{\beta}^{(r)} + a_r \delta_r \hat{\beta}^{(r)}$$

- Set $a_r = 1/\sqrt{A}$, then

$$\hat{V}(\hat{t}_{y,\text{reg}}) = A \sum_{r=1}^{R} \left( \hat{t}_{y,\text{reg}}^{(r)} - \hat{t}_{y,\text{reg}} \right)^2$$

consistent for

$$\text{AVar}(\hat{t}_{y,\text{reg}}) = \text{Var}(\hat{t}_e) + \beta_U^T \text{Var}(\hat{t}_{Cx}) \beta_U$$
Implementing Fuller (1998) Method (2)

What if the numbers of control totals and replicates differ?

- If $H \leq R$:
  \[
  a_r = \begin{cases} 
  \frac{1}{\sqrt{A}} & r = 1, \ldots, H \\
  0 & r = H + 1, \ldots, R 
  \end{cases}
  \]

- If $H > R$: use $\delta_h$ corresponding to $R$ largest eigenvalues, which assumes that
  \[
  \sum_{r=1}^{R} \delta_h \delta_h^T \approx \hat{V}(\hat{t}_{Cx})
  \]
  (low-rank approximation)

Works for calibration by regression, post-stratification and raking
3. Application to APAIS Calibration

- 2019 APAIS and FES datasets
- Post-stratify APAIS weights to match FES estimated trip totals for 16 states, 2 modes (shore, private boat), 6 waves
  ⇒ 160 control estimates ($R = H$)
3. Application to APAIS Calibration (2)

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3. Application to APAIS Calibration (3)

- Investigate scenario when $H > R$
- 172 FES controls: estimated trip totals for 17 “states,” 2 modes (shore, private boat), 6 waves
- Use $\delta_h$ of 160 largest eigenvalues of $\hat{V}(\hat{t}_{Cx})$
3. Application to APAIS Calibration (4)

- **Distribution of CVs over 172 calibration domains**

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- **Distribution of CVs over 160 calibration domains**

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4. Conclusions

- Sample-based calibration can be very useful in practice, e.g.
  - organization conducts multiple surveys and wishes to report consistent estimates
  - following changes in survey methodology, survey results are no longer comparable with previous surveys and need to be adjusted
  - fixed controls are not available
  - multi-phase samples

- Important to reflect calibration to sample-based controls in measures of precision

- Can be accomplished easily within replication methods for primary survey

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