Simultaneous Edit-Imputation for Categorical Microdata

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2013 FCSM Research Conference
November 6th, 2013

Research supported by NSF grant SES-11-31897.

The problem

Inconsistent Datasets

- Many individual level multivariate datasets, e.g. surveys, have consistency requirements specifying combinations of responses that are not allowed.
- In real-life, however, datasets often include errors.
  - When the errors end up in a violation of a consistency rule, we can detect the error.
  - When the error doesn't result in a consistency rule violation, the error is not detectable.

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We Want

1. Detect and locate errors (even if they don't result in the violation of a consistency rule.)
2. Impute consistent values, respecting the distribution the data, and reflecting the uncertainty associated with the procedure.

Conceptualizing the Problem

- Data consists of vectors $\mathbf{Y}_i = (Y_{i1}, ..., Y_{iJ})$, $i = 1, ..., n$ (e.g. *recorded* responses to J survey questions)
- Each of the J components take values from a finite set $Y_{ij} \in \{1, 2, ..., L_j\}$.
- Entries in $\mathbf{Y}_i$ might be inconsistent. Then $\mathbf{Y}_i \in \mathcal{C} = \prod_{j=1}^{J} \{1, ..., L_j\}$.
- Consistency rules are a collection of $S \subsetneq \mathcal{C}$ that specify which values of $\mathbf{Y}_i$ shouldn't be present in the dataset.
- Connections to structural zeros in contingency tables.

A Generative Perspective

- The observed response $\mathbf{Y}_i$ is a contaminated version of a "true" underlying response, $\mathbf{X}_i$.
- $\mathbf{Y}_i$ is observed. $\mathbf{X}_i$ is unobserved.
- $\Pr(\mathbf{Y}_i \in S) > 0$. $\Pr(\mathbf{X}_i \in S) = 0$.
- We assume a generation process for $\mathbf{X}_i$

  $$\mathbf{X}_i \overset{iid}{\sim} F,$$

  which doesn't allow for inconsistent values. $\mathbf{X}_i \in \mathcal{C} \setminus S$.
- $\mathbf{Y}_i$s come from an "error process"

  $$\mathbf{Y}_i | \mathbf{X}_i \sim E(\mathbf{X}_i).$$

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Our objective is to estimate F.

Error models

- Given true data, the error process determines what we observe.
- We differentiate two components:
  1. **Location model:** Which items are in error?
  2. **Substitution model:** Given that there's an error at the (i, j) location, how does Y_{ij} is generated from X_{ij}?

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- Let $E_{ij} = 1$ if there's an error at the (i, j) location, and 0 otherwise. We define the *error mask* $\mathbf{E}_i = (E_{i1}, ..., E_{iJ}) \in \{0, 1\}^J$.

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- The **location model** is the distribution of $\mathbf{E}_i$.
- The **substitution model** is the conditional distribution of $\mathbf{Y}_i$ given $\mathbf{E}_i$ and $\mathbf{X}_i$
- (This separation allows to specify a priori which values we *know* are correct or incorrect.)

Specifying the Error Model

Location: Independent Errors Model

$$E_{ij}|\epsilon_j \overset{indep}{\sim} \text{Bernoulli}(\epsilon_j)$$
$$\epsilon_j \overset{iid}{\sim} \text{Beta}(a_\epsilon, b_\epsilon)$$

- Error locations are independent.
- Each item has its own error rate, ϵ_j.
- Other specifications possible.

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Substitution: Uniform Substitution Model

$$Y_{ij}|X_{ij}, E_{ij} \sim \begin{cases} \delta_{X_{ij}} & \text{if } E_{ij} = 0 \\ \text{Uniform}\left(\{1, ..., L_j\} \setminus \{X_{ij}\}\right) & \text{if } E_{ij} = 1 \end{cases}$$

Data Generation Models

"True Responses" Distribution

$$\mathbf{X}_i \sim F$$

- In principle it can be any distribution over $\mathcal{C} \setminus S$.
- In practice we need a flexible enough specification, able to capture the nuances of the multivariate structure.
- Challenges:
  - Sparsity (very high-dimensional tables with many zero-counts).
  - Model selection. We want high prediction power.
  - Handling of structural zeros!

We use the Nonparametric Truncated Latent Class Model from Manrique-Vallier and Reiter, 2013 (JCGS, to appear)

Truncated mixtures of discrete distributions:

$$\mathbf{x}_i | \boldsymbol{\lambda}, \boldsymbol{\pi} \sim 1\{\mathbf{x}_i \notin S\} \sum_{k=1}^{\infty} \pi_k \prod_{j=1}^{J} \lambda_{jk(x_{ij})}$$

with $\boldsymbol{\pi} = (\pi_1, \pi_2, ...) \sim DP(\alpha)$, $\lambda_{jk} \stackrel{iid}{\sim} Dirichlet(\mathbf{1}_K)$, and $\alpha \sim Gamma(a_\alpha, b_\alpha)$.

- Very flexible models.
- Method by Manrique-Vallier and Reiter (2013) to obtain posterior parameter samples subject to truncated (to $\mathcal{C} \setminus S$) data support.
- Several advantages: Automatic overfitting control. Computationally tractable. High tolerance to sparsity. Capacity to handle large collections of structural zeros.

Test Application - Data Based Simulation

$J = 10$ variables from 5% public use microdata from 2000 U.S. census (NY)

Variable	Levels (L_j)	Variable	Levels (L_j)
Ownership of dwelling	3	Mortgage status	4
Age	9	Sex	2
Marital status	6	Race	5
Education	11	Employment	4
Work disability	3	Veteran Status	3

- Take $N = 953,076$ as a population. Compute statistics.
- Sub-sample $n = 1,000$, introduce errors, fix them, and try to estimate population quantities back.

Notes:

- Resulting contingency table has $2,566,080$ cells.
- $|S| = 2,317,030$ possible inconsistent responses. Originally specified as 60 pair-wise rules (e.g. veteran toddlers).
- Original data without inconsistencies.

Test Application - Introducing Errors

Contaminate the data using independent errors and uniform substitution,

$$Y_{ij} | X_{ij}, E_{ij} \sim \begin{cases} \delta_{X_{ij}} & \text{if } E_{ij} = 0 \\ \text{Uniform}\left(\{1, ..., L_j\} \setminus \{X_{ij}\}\right) & \text{if } E_{ij} = 1 \end{cases}$$

$$E_{ij} \overset{iid}{\sim} \text{Bernoulli}(\varepsilon)$$

- Try with different error rates $\varepsilon = 0.1, 0.3, 0.5$.
- Pretend that we only observe **Y**.

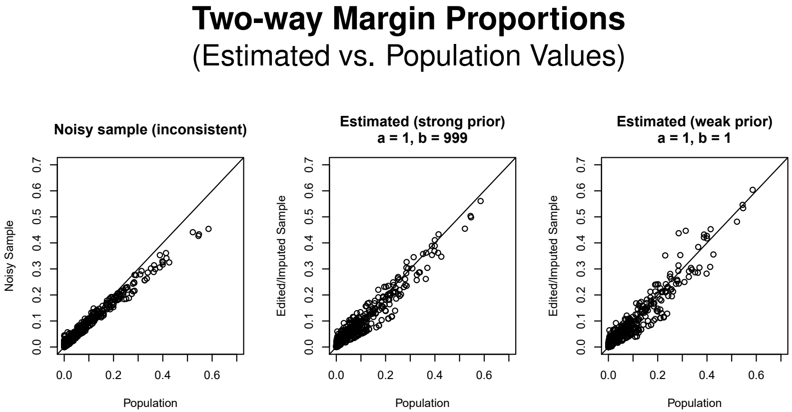
Prior Specification for Error Model

- We use the independent errors / uniform substitution model.
- Need to specify prior distribution for item error rates:

$$\epsilon_j \sim Beta(a_\epsilon, b_\epsilon)$$

- The method will always detect and correct *detectable* errors.
- The prior specification determines how much we trust what we observe:
  - a_ϵ / b_ϵ = Prior expected rate of error.
  - Large $a_\epsilon + b_\epsilon$ (relative to sample size) puts more weight on our beliefs than on the data.
  - Small $a_\epsilon + b_\epsilon$ puts more weight on data.
- For variables that we don't want to ever alter, we set $E_{ij} = 0$ a priori. This forces $Y_{ij} = X_{ij}$. (can have unintended consequences, though)

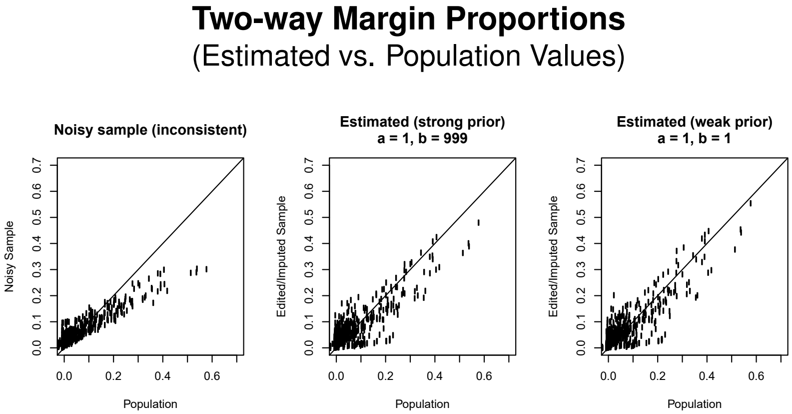
Results (1)- Two-Way margins (ε 0.1)



Simulation Parameters:

- $\varepsilon = 0.1, n = 1,000$
- Rows with errors = 626. Detectable errors = 306

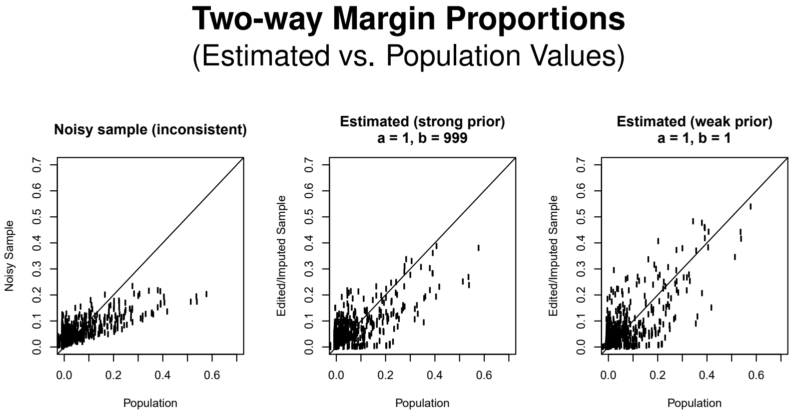
Results (2)- Two-Way margins (ε 0.3)



Simulation Parameters:

- $\varepsilon = 0.3, n = 1,000$
- Rows with errors = 980. Detectable errors = 685

Results (3)- Two-Way margins (ε 0.5)



Simulation Parameters:

- $\varepsilon = 0.5, n = 1,000$
- Rows with errors = 999. Detectable errors = 833

Concluding Remarks

- Full Bayesian model-based approach to edit-imputation.
- Integrates data generation with measurement error.
- Automatic over-fitting protection.
- Edit and imputation based on joint distribution. Respects data distribution.
- Does not require full analysis of consistency rules. Guaranteed to generate consistent imputations.
- Computationally feasible, but can be demanding in tough problems. (runtime example = 1.6 min)
- Prior specification matters:
  - Strong prior w/low error rate.
  - Weak prior.
- Open issue: Which values do we really want to change? (prior for ϵ_j and which E_{ij} set to 0 a priori)

The End

(Thanks!)

For details about truncated latent structure models:

`http://mypage.iu.edu/~dmanriqu/papers/lcm_zeros.pdf`

For multiple imputation see:

`http://mypage.iu.edu/~dmanriqu/papers/LCM_Zeros_Imputation.pdf`