Simultaneous Edit-Imputation for Categorical Microdata

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2013 FCSM Research Conference
November 6th, 2013

Research supported by NSF grant SES-11-31897.
Inconsistent Datasets

- Many individual level multivariate datasets, e.g. surveys, have consistency requirements specifying combinations of responses that are not allowed.
- In real-life, however, datasets often include errors.
  - When the errors end up in a violation of a consistency rule, we can detect the error.
  - When the error doesn’t result in a consistency rule violation, the error is not detectable.
The problem

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We Want

1. Detect and locate errors (even if they don’t result in the violation of a consistency rule.)
2. Impute consistent values, respecting the distribution the data, and reflecting the uncertainty associated with the procedure.
Conceptualizing the Problem

- Data consists of vectors $\mathbf{Y}_i = (Y_{i1}, \ldots, Y_{iJ})$, $i = 1, \ldots, n$ (e.g. recorded responses to $J$ survey questions).

- Each of the $J$ components take values from a finite set $Y_{ij} \in \{1, 2, \ldots, L_j\}$.

- Entries in $\mathbf{Y}_i$ might be inconsistent. Then $\mathbf{Y}_i \in \mathcal{C} = \prod_{j=1}^{J} \{1, \ldots, L_j\}$.

- Consistency rules are a collection of $S \subsetneq \mathcal{C}$ that specify which values of $\mathbf{Y}_i$ shouldn't be present in the dataset.

- Connections to structural zeros in contingency tables.
A Generative Perspective

- The observed response $Y_i$ is a contaminated version of a “true” underlying response, $X_i$.
- $Y_i$ is observed. $X_i$ is unobserved.
- $\Pr(Y_i \in S) > 0$. $\Pr(X_i \in S) = 0$.
- We assume a generation process for $X_i$

$$X_i \overset{iid}{\sim} F,$$

which doesn’t allow for inconsistent values. $X_i \in C \setminus S$.

- $Y_i$s come from an “error process”

$$Y_i|X_i \sim E(X_i).$$

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Our objective is to estimate $F$. 
Given true data, the error process determines what we observe.

We differentiate two components:

1. **Location model**: Which items are in error?
2. **Substitution model**: Given that there’s an error at the \((i, j)\) location, how does \(Y_{ij}\) is generated from \(X_{ij}\)?
Error models

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  1. **Location model**: Which items are in error?
  2. **Substitution model**: Given that there’s an error at the \((i, j)\) location, how does \(Y_{ij}\) is generated from \(X_{ij}\)?
- Let \(E_{ij} = 1\) if there’s an error at the \((i, j)\) location, and 0 otherwise. We define the *error mask* \(E_i = (E_{i1}, \ldots, E_{ij}) \in \{0, 1\}^J\).
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\[ E_i = (E_{i1}, \ldots, E_{iJ}) \in \{0, 1\}^J. \]

The **location model** is the distribution of \(E_i\).

The **substitution model** is the conditional distribution of \(Y_i\) given \(E_i\) and \(X_i\).

(This separation allows to specify a priori which values we know are correct or incorrect.)
Location: Independent Errors Model

\[ E_{ij} | \epsilon_j \overset{indep}{\sim} \text{Bernoulli}(\epsilon_j) \]
\[ \epsilon_j \overset{iid}{\sim} \text{Beta}(a_{\epsilon}, b_{\epsilon}) \]

- Error locations are independent.
- Each item has its own error rate, \( \epsilon_j \).
- Other specifications possible.
Specifying the Error Model

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**Substitution: Uniform Substitution Model**

\[ Y_{ij} | X_{ij}, E_{ij} \sim \begin{cases} 
\delta X_{ij} & \text{if } E_{ij} = 0 \\
\text{Uniform } \left( \{1, \ldots, L_j\} \setminus \{X_{ij}\} \right) & \text{if } E_{ij} = 1 
\end{cases} \]
“True Responses” Distribution

\[ X_i \sim F \]

- In principle it can be any distribution over \( C \setminus S \).
- In practice we need a flexible enough specification, able to capture the nuances of the multivariate structure.
- Challenges:
  - Sparsity (very high-dimensional tables with many zero-counts).
  - Model selection. We want high prediction power.
  - Handling of structural zeros!

We use the Nonparametric Truncated Latent Class Model from Manrique-Vallier and Reiter, 2013 (JCGS, to appear)
Truncated mixtures of discrete distributions:

\[ x_i | \lambda, \pi \sim 1\{x_i \notin S\} \sum_{k=1}^{\infty} \pi_k \prod_{j=1}^{J} \lambda_{jk}(x_{ij}) \]

with \( \pi = (\pi_1, \pi_2, \ldots) \sim DP(\alpha), \lambda_{jk} \overset{iid}{\sim} Dirichlet(1_K), \) and \( \alpha \sim Gamma(a_\alpha, b_\alpha). \)

- Very flexible models.
- Method by Manrique-Vallier and Reiter (2013) to obtain posterior parameter samples subject to truncated (to \( C \setminus S \)) data support.
$J = 10$ variables from 5% public use microdata from 2000 U.S. census (NY)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Levels ($L_j$)</th>
<th>Variable</th>
<th>Levels ($L_j$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ownership of dwelling</td>
<td>3</td>
<td>Mortgage status</td>
<td>4</td>
</tr>
<tr>
<td>Age</td>
<td>9</td>
<td>Sex</td>
<td>2</td>
</tr>
<tr>
<td>Marital status</td>
<td>6</td>
<td>Race</td>
<td>5</td>
</tr>
<tr>
<td>Education</td>
<td>11</td>
<td>Employment</td>
<td>4</td>
</tr>
<tr>
<td>Work disability</td>
<td>3</td>
<td>Veteran Status</td>
<td>3</td>
</tr>
</tbody>
</table>

- Take $N = 953,076$ as a population. Compute statistics.
- Sub-sample $n = 1,000$, introduce errors, fix them, and try to estimate population quantities back.

Notes:
- Resulting contingency table has 2,566,080 cells.
- $|S| = 2,317,030$ possible inconsistent responses. Originally specified as 60 pair-wise rules (e.g. veteran toddlers).
- Original data without inconsistencies.
Contaminate the data using independent errors and uniform substitution,

\[ Y_{ij}|X_{ij}, E_{ij} \sim \begin{cases} 
\delta X_{ij} \\
\text{Uniform} \left( \{1, \ldots, L_j\} \setminus \{X_{ij}\} \right)
\end{cases} \quad \text{if } E_{ij} = 0 \]
\[ E_{ij} \overset{iid}{\sim} \text{Bernoulli}(\varepsilon) \quad \text{if } E_{ij} = 1 \]

- Try with different error rates \( \varepsilon = 0.1, 0.3, 0.5. \)
- Pretend that we only observe \( Y. \)
We use the independent errors / uniform substitution model.

Need to specify prior distribution for item error rates:

\[ \epsilon_j \sim \text{Beta}(a_\epsilon, b_\epsilon) \]

The method will always detect and correct detectable errors.

The prior specification determines how much we trust what we observe:

- \( a_\epsilon / b_\epsilon \) = Prior expected rate of error.
- Large \( a_\epsilon + b_\epsilon \) (relative to sample size) puts more weight on our beliefs than on the data.
- Small \( a_\epsilon + b_\epsilon \) puts more weight on data.

For variables that we don’t want to ever alter, we set \( E_{ij} = 0 \) a priori. This forces \( Y_{ij} = X_{ij} \). (can have unintended consequences, though)
Results (1) - Two-Way margins ($\varepsilon = 0.1$)

Two-way Margin Proportions
(Estimated vs. Population Values)

Simulation Parameters:
- $\varepsilon = 0.1$, $n = 1,000$
- Rows with errors = 626. Detectable errors = 306
Two-way Margin Proportions
(Estimated vs. Population Values)

Simulation Parameters:
- $\varepsilon = 0.3$, $n = 1,000$
- Rows with errors = 980. Detectable errors = 685
Results (3)- Two-Way margins ($\varepsilon = 0.5$)

**Two-way Margin Proportions**
(Estimated vs. Population Values)

Simulation Parameters:
- $\varepsilon = 0.5$, $n = 1,000$
- Rows with errors = 999. Detectable errors = 833
Concluding Remarks

- Full Bayesian model-based approach to edit-imputation.
- Integrates data generation with measurement error.
- Automatic over-fitting protection.
- Edit and imputation based on joint distribution. Respects data distribution.
- Does not require full analysis of consistency rules. Guaranteed to generate consistent imputations.
- Computationally feasible, but can be demanding in tough problems. (runtime example = 1.6 min)
- Prior specification matters:
  - Strong prior w/low error rate.
  - Weak prior.
- Open issue: Which values do we really want to change? (prior for $\epsilon_j$ and which $E_{ij}$ set to 0 a priori)
The End
(Thanks!)

For details about truncated latent structure models:

For multiple imputation see: