Using Gaussian copula to generate a synthetic population

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Outline

- Purpose of research
- Census of Agriculture
- Gaussian copula
- R package dplyr and ggplot2
  - Comparing dplyr and ggplot2 with base R
  - Generating synthetic population using dplyr and ggplot2
- Conclusion

Purpose of presentation

- Generating a complex synthetic population
    - To protect confidential information provided by responders
    - To maintain pairwise statistical relationships among variables
    - To handle continuous variable and categorical variable simultaneously

- Introducing statistical R packages ggplot2, dplyr that are used for visualization, data processing respectively

Census of Agriculture

- Every five years, USDA's National Agricultural Statistics Service (NASS) conducts the Census of Agriculture
  - The Census provides a detailed picture of U.S. farms, ranches and the people who operate them
  - It is the only source of uniform, comprehensive agricultural data for every state and county in the United States
  - NASS also obtains information on most commodities from administrative sources or surveys of non-farm populations (e.g. cotton ginning data)

Census of Agriculture data overview

- Generate a synthetic population based on a subset of the Census of Agriculture data
  - The subset contains 25 predictors
    - For example, principal operator’s race, principal operator’s sex, etc…, but they are relabeled as X1, …, X25
  - There are continuous and categorical variables
  - The total number of observations in the subset is 800,000
  - No missing value

Gaussian copula

- Copulas are used to describe the dependence among random variables
- A copula is a multivariate probability distribution for which the marginal probability distribution of each variable is uniform
  - The marginal CDFs $F_i(x)$ of a random vector *X*$(X_1, X_2, \ldots, X_d)$ follows a uniform distribution U_i
  - The copula is defined as

$$C(u_1, u_2, \ldots, u_d) = P\,(U_1 \le u_1, U_2 \le u_2, \ldots, U_d \le u_d) = P(F_1^{-1}(u_1), F_2^{-1}(2) \ldots, F_d^{-1}(u_d))$$

- Gaussian copula is constructed from a multivariate normal distribution with correlation matrix *P*:

$$C_P(u_1, u_2, \ldots, u_d) = \Phi_P(\Phi_1^{-1}(u_1), \Phi_2^{-1}(2) \ldots, \Phi_d^{-1}(u_d))$$

where Φ denotes the standard normal distribution function, and Φ_P denotes the multivariate standard normal distribution function with correlation matrix *P*

Gaussian copula applied in generating synthetic population

- Perform a cholesky decomposition of correlation matrix *P*, and set *A* as the resulting lower triangular matrix

- Repeat the following steps *n* times
  - Generate a vector $Z = (Z_1, \ldots, Z_d)'$ of independent standard normal deviates
  - Set $X = AZ$
  - Return $U = (\Phi(X_1), \ldots, \Phi(X_d))'$

- Can be achieved using *mvrnorm* in *MASS* library

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R packages introduction

- A Grammar of Data Manipulation (dplyr)
  - R-package used for data processing
  - Transform and summarize tabular data with rows and columns.
  - Contain a set of functions (or “verbs”) that perform common data manipulation operations
- Create Elegant Data Visualizations Using the Grammar of Graphics (ggplot2)
  - R-package used for data visualization
  - Consistent underlying grammar of graphics (a graphic version of dplyr)
  - Plot specification at a high level of abstraction
  - Very flexible and elegant

R packages introduction - dplyr

- Commonly used command:
  - mutate() adds new variables that are functions of existing variables
  - select() picks variables based on their names
  - filter() picks cases based on their values
  - summarise() reduces multiple values down to a single summary
  - arrange() changes the ordering of the rows
  - group_by() allows for group operations in the “split-apply-combine” concept

R packages Introduction – dplyr - Continue

- Base R:
  - Create new dataset:

```
pop_census$new1 <- pop_census$X10 + 0.5 * pop_census$X11
pop_census$new2 <- pop_census$X13 + 3 * pop_census$X14
pop_census$new3 <- log(pop_census$X15)
pop_census$new4 <- pop_census$X16 * 4
```

  - Select Variable:

```
pop_census[ ,c('X14','X15', 'X16', 'new1', 'X18')]
```

  - Filter variable:

```
pop_census [pop_census$X10 >=3 & pop_census$X16 >=1 & pop_census$X17 <=7,]
```

  - Sort by variable value:

```
pop_census[ order(pop_census$X19),]
```

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R packages Introduction – dplyr - Continue

- dplyr
  - This can be done with one function using dplyr:

```
pop_census <- pop_census %>% mutate(new1 = X10 + 0.5 * X11,
                                    new2 = X13 + 3 * X14,
                                    new3 = log(X15),
                                    new4 = X16 * 4) %>%
                             dplyr::select (X14 , X15, X16, new1, X18)  %>%
                             filter (X10 >=1, X16 >=1 , X17 <=7) %>%
                             arrange(X19)
```

```
pop_census %>%
  group_by(X2) %>%
  summarise(avg_X15 = mean(X15),
            min_X15 = min(X15),
            max_X15 = max(X15),
            total = n())
```

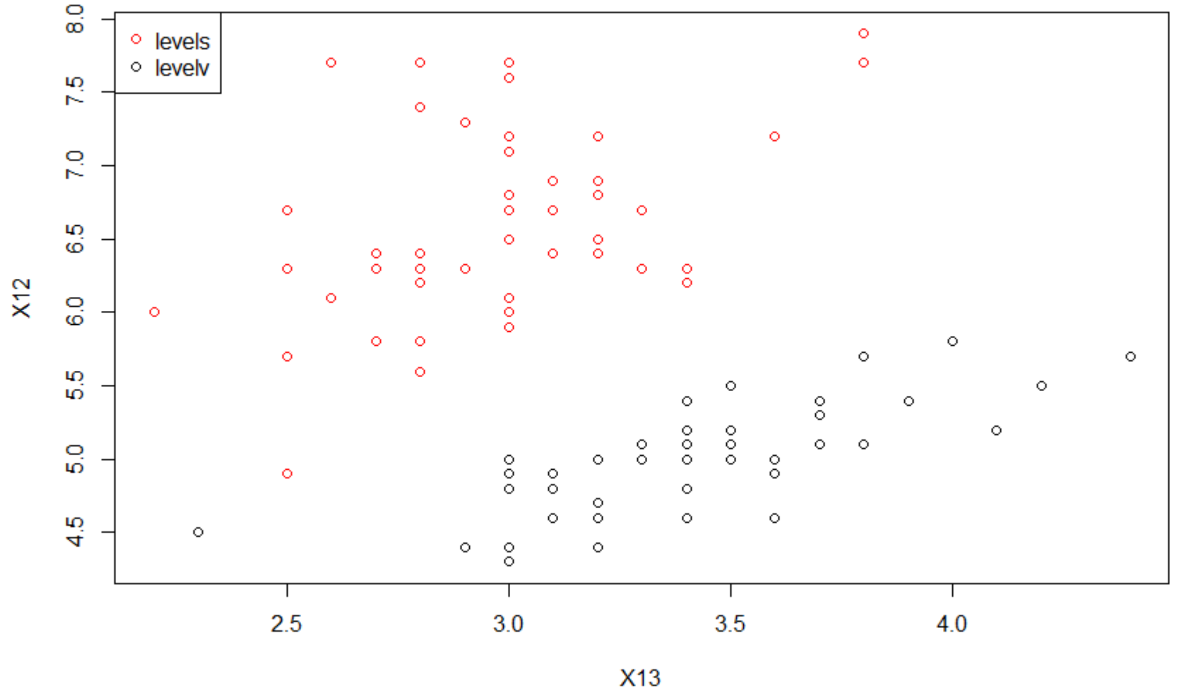
R packages introduction – ggplot2

- Graphic version of dplyr (using ‘+’ to replace ‘%>%’)
- Building blocks of a graph include:
  - data
  - aesthetic mapping
  - geometric object
  - statistical transformations
  - scales
  - coordinate system
  - position adjustments
  - faceting

R packages introduction – ggplot2 - Continue

- base R

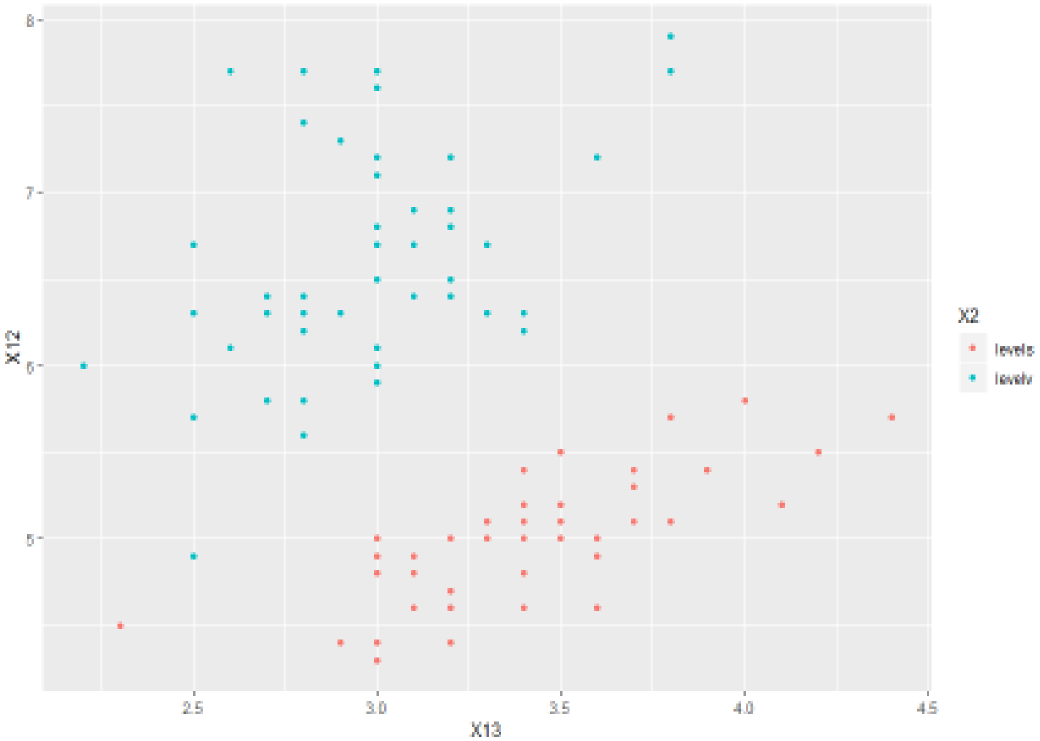
```
plot(X12 ~ X13,
     col = factor(X2),
     data = filter(pop_census, X2 %in% c("levels", "levelv")))
legend("topleft",
       legend = c("levels", "levelv"),
       col = c("red", "black"),
       pch = 1)
```

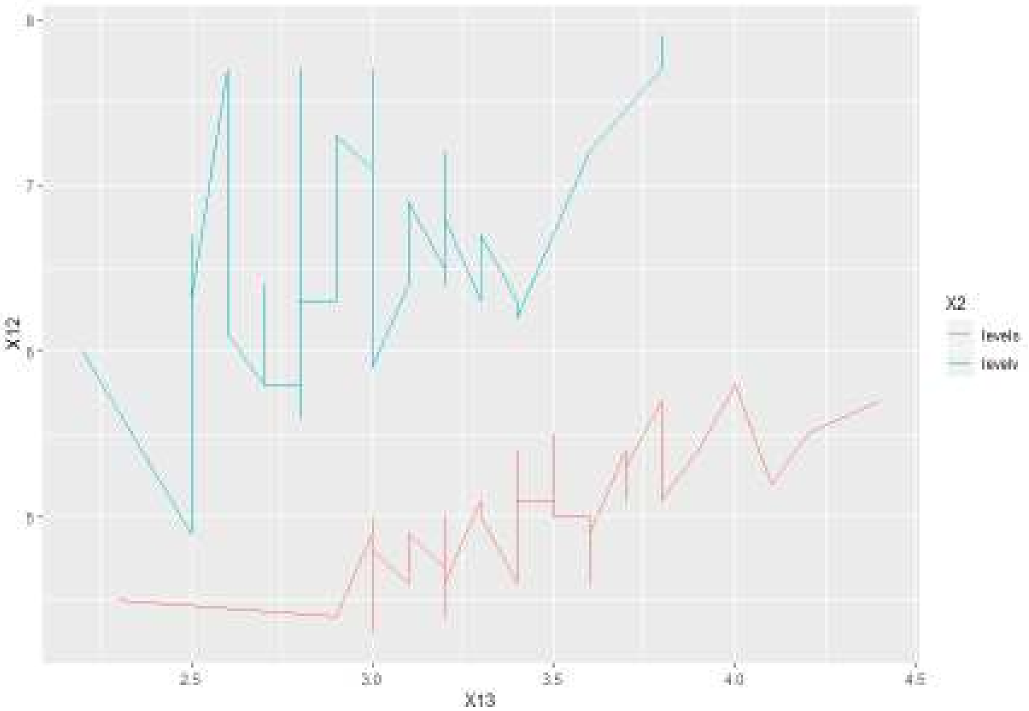


R packages Introduction – dplyr - Continue

- ggplot2

```
ggplot(filter(pop_census, X2 %in% c("levels", "levelv")),
           aes(x=X13,
                y=X12,
                color=X2))+
   geom_point()
```





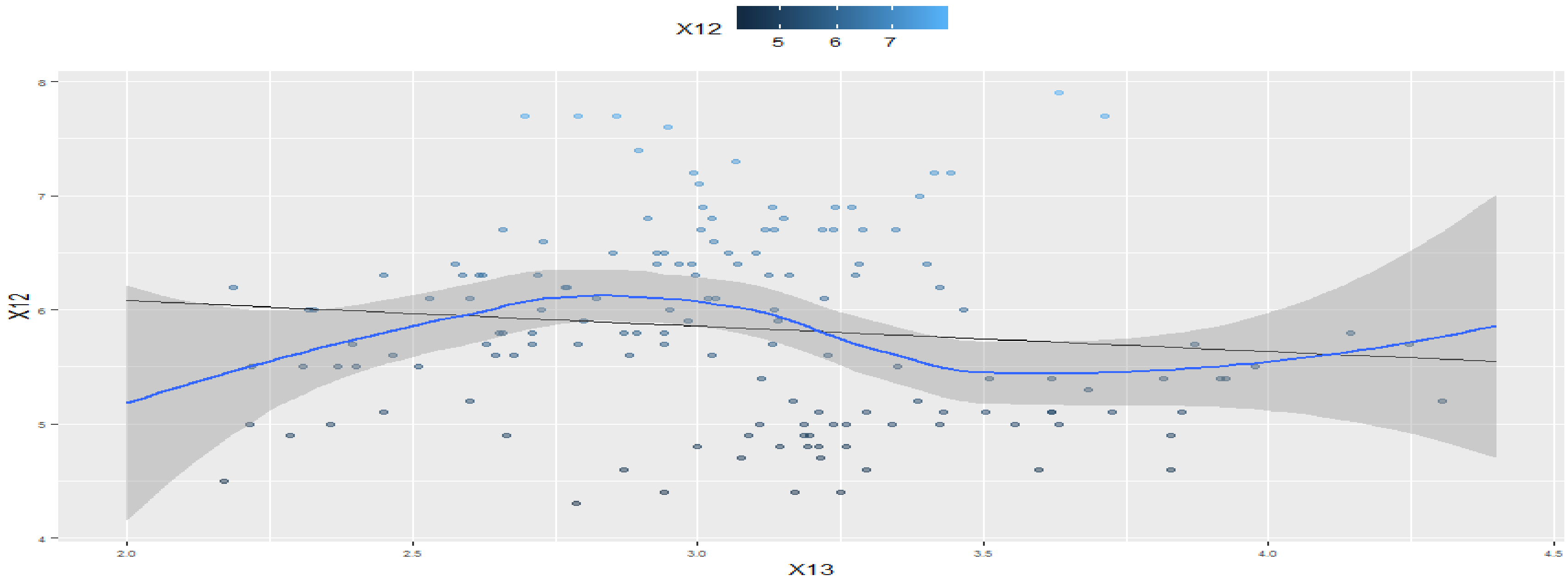
R packages Introduction – ggplot2 - Continue

- More advanced features

```
pop_census$pred.X12 <- predict(lm(X12 ~ X13, data = pop_census))

p1 <- ggplot(pop_census, aes(x = X13, y = X12)) +  theme(legend.position="top",
                                                          axis.text=element_text(size = 6))

p1 + geom_point(aes(color = X12),alpha = 0.5,
                size = 1.5,
                position = position_jitter(width = 0.25, height = 0)) +
  geom_line(aes(y = pred.X12)) +   geom_smooth()
```



Step by step R function

- Categorical variables are converted to continuous before using copula
  - The frequency of the categorical variables' levels are used as the value for that level

```
dat[,cat] = apply(dat[,cat],2,function(x){
                t = as.data.frame(table(x))
                t$Freq[match(x,t[,1])]/length(x)
              })
```

- CDF of the variable are transformed to be used in copula
  - U_i is transformed to $\Phi_i^{-1}(u_i)$ using *ecdf* and *qnorm*

```
prepare_copula_qnorm <- function (var){
  #This function is designed to calculate the copula
  new_var = qnorm(ecdf(var)(var)*0.99 + 0.005)
  return (new_var)
}
```

Step by step R function - Continue

```
dat <- dat %>% mutate (New_X1 = case_when( X1 == 1 ~ 1,
                                           X1 ==1 ~ 2,
                                         TRUE~ 0 ),
                       New_X2 = case_when((X2>=9 & X2 <=16) ~ 2,
                                     TRUE ~ 1),
                       New_X3= case_when(X3<=9 ~ 1,
                                   (X3>=10 & X3 <=49) ~ 2,
                                   (X3>=50 & X3 <=69) ~ 3,
                                   (X3>=70 & X3 <=99) ~ 4,
                                   (X3>=100 & X3 <=139) ~ 5,
                                   (X3>=140 & X3 <=179) ~ 6,
                                   (X3>=180 & X3 <=219) ~ 7,
                                   (X3>=220 & X3 <=259) ~ 8,
                                   (X3>=260 & X3 <=499) ~ 9,
                                   (X3>=500 & X3 <=999) ~ 10,
                                   (X3>=1000 & X3 <=1999) ~ 11,
                                   (X3>=2000) ~ 12))
dat[,cat] = apply(dat[,cat],2,function(x){
                t = as.data.frame(table(x))
                t$Freq[match(x,t[,1])]/length(x)
              })
```

Step by step R function - Continue

```
  dat <- dat %>% mutate (
                        New_X1_continuous = prepare_copula_qnorm(dat$New_X1),
                        New_X2_continuous= prepare_copula_qnorm(dat$New_X2),
                        New_X3_continuous = prepare_copula_qnorm(dat$New_X3),
                        X4_continuous = prepare_copula_qnorm(dat$X4),X5_continuous= prepare_copula_qnorm(dat$X5),
                        X6_continuous = prepare_copula_qnorm(dat$X6),X7_continuous= prepare_copula_qnorm(dat$X7),
                        X8_continuous = prepare_copula_qnorm(dat$X8),X9_continuous= prepare_copula_qnorm(dat$X9),
                        X10_continuous = prepare_copula_qnorm(dat$X10),X11_continuous= prepare_copula_qnorm(dat$X11),
                        X12_continuous = prepare_copula_qnorm(dat$X12),X13_continuous= prepare_copula_qnorm(dat$X13),
                        X14_continuous = prepare_copula_qnorm(dat$X14),X15_continuous= prepare_copula_qnorm(dat$X15),
                        X16_continuous = prepare_copula_qnorm(dat$X16),X17_continuous= prepare_copula_qnorm(dat$X17),
                        X18_continuous = prepare_copula_qnorm(dat$X18),X19_continuous= prepare_copula_qnorm(dat$X19),
                        X20_continuous = prepare_copula_qnorm(dat$X20),X21_continuous= prepare_copula_qnorm(dat$X21),
                        X22_continuous = prepare_copula_qnorm(dat$X22),X23_continuous= prepare_copula_qnorm(dat$X23),
                        X24_continuous = prepare_copula_qnorm(dat$X24),X25_continuous= prepare_copula_qnorm(dat$X25)
                        )
  return (dat)
}
```

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Step by step R function - Continue

```
create_newpop_sizetype<- function(m,n, pop_num, dat_tol){
  #This function is designed to create synthetic population within each New_X2 and New_X3
  dat_continuous <- dat_tol %>%
    filter (New_X2 ==m, New_X3==n) %>%
    dplyr::select(New_X1_continuous,  X4_continuous, X5_continuous,
                  X6_continuous, X7_continuous, X8_continuous, X9_continuous, X10_continuous,
                  X11_continuous, X12_continuous, X13_continuous, X14_continuous, X15_continuous,
                  X16_continuous, X17_continuous, X18_continuous, X19_continuous, X20_continuous,
                  X21_continuous, X22_continuous, X23_continuous, X24_continuous, X25_continuous)
  dat_cor <- cor(dat_continuous)
  #generating new population
  new_pop <- mvrnorm(n = pop_num, mu = rep(0,nrow(dat_cor)), dat_cor, tol = 1e-6, empirical = FALSE, EISPACK = FALSE)
  return (new_population)
}
```

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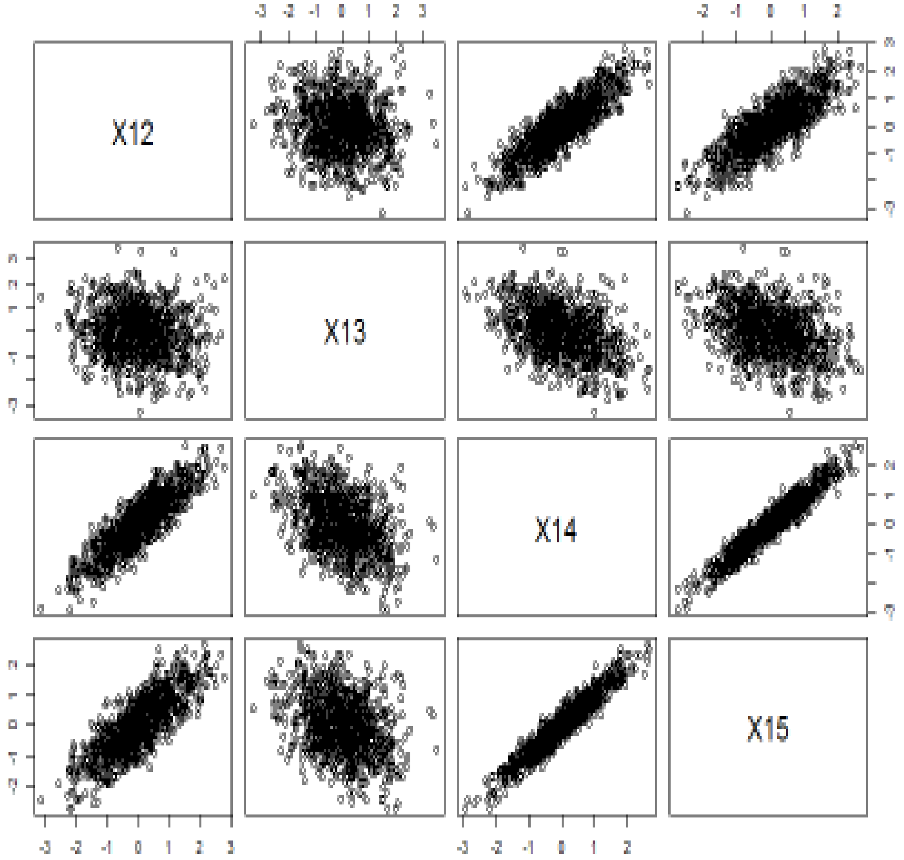
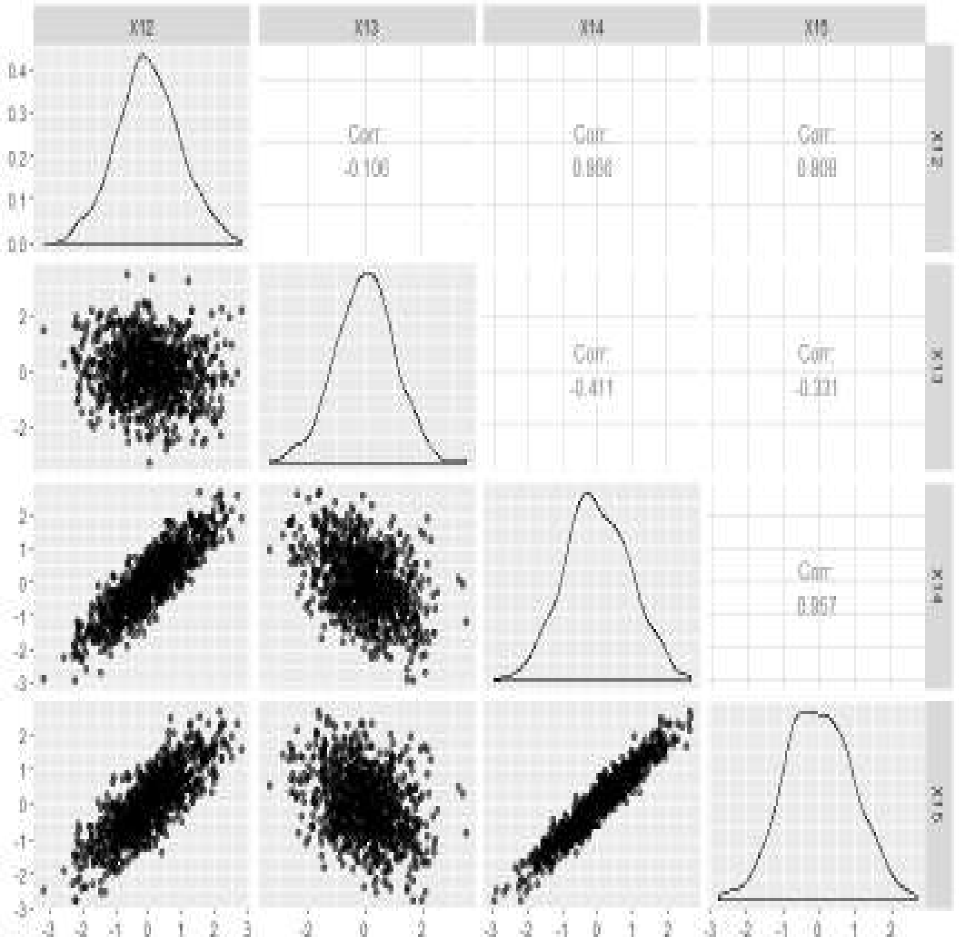
Pairwise correlation comparison





Summary

- This study
  - Generating synthetic population by Gaussian copula
  - Adopting two R packages dplyr and ggplot2 simplifed the tasks for this study

- dplyr and ggplot2 are two useful R packages
  - More convenient to manage
  - Easy to read
  - Decrease the workload
  - My personal preference to use over base R

References

Nelsen, R. B. (2007). *An introduction to copulas*. Springer Science & Business Media.

Wickham, H., Francois, R., Henry, L., & Müller, K. (2015). dplyr: A grammar of data manipulation. *R package version 0.4, 3*.

Wickham, H. (2016). *ggplot2: elegant graphics for data analysis*. Springer.

Any Questions?

Thank you!