Using Gaussian copula to generate a synthetic population

Yijun Wei
NISS, USDA-NASS
Yijun.Wei@nass.usda.gov

Luca Sartore
NISS, USDA-NASS
Luca.Sartore@nass.usda.gov

Nell Sedransk
NISS
NSedransk@niss.org
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Outline

• Purpose of research
• Census of Agriculture
• Gaussian copula
• R package dplyr and ggplot2
  • Comparing dplyr and ggplot2 with base R
  • Generating synthetic population using dplyr and ggplot2
• Conclusion
Purpose of presentation

• Generating a complex synthetic population
  • To protect confidential information provided by responders
  • To maintain pairwise statistical relationships among variables
  • To handle continuous variable and categorical variable simultaneously

• Introducing statistical R packages ggplot2, dplyr that are used for visualization, data processing respectively
Census of Agriculture

• Every five years, USDA's National Agricultural Statistics Service (NASS) conducts the Census of Agriculture
  • The Census provides a detailed picture of U.S. farms, ranches and the people who operate them
  • It is the only source of uniform, comprehensive agricultural data for every state and county in the United States
  • NASS also obtains information on most commodities from administrative sources or surveys of non-farm populations (e.g. cotton ginning data)
Census of Agriculture data overview

• Generate a synthetic population based on a subset of the Census of Agriculture data
  • The subset contains 25 predictors
    • For example, principal operator’s race, principal operator’s sex, etc..., but they are relabeled as X1, ..., X25
  • There are continuous and categorical variables
  • The total number of observations in the subset is 800,000
  • No missing value
Copulas are used to describe the dependence among random variables

A copula is a multivariate probability distribution for which the marginal probability distribution of each variable is uniform

- The marginal CDFs $F_i(x)$ of a random vector $X(X_1, X_2, \ldots, X_d)$ follows a uniform distribution $U_i$
- The copula is defined as

$$C(u_1, u_2, \ldots, u_d) = P(U_1 \leq u_1, U_2 \leq u_2, \ldots, U_d \leq u_d) = P(F_1^{-1}(u_1), F_2^{-1}(2) \ldots, F_d^{-1}(u_d))$$

- Gaussian copula is constructed from a multivariate normal distribution with correlation matrix $P$:

$$C_P(u_1, u_2, \ldots, u_d) = \Phi_P(\Phi_1^{-1}(u_1), \Phi_2^{-1}(2) \ldots, \Phi_d^{-1}(u_d))$$

where $\Phi$ denotes the standard normal distribution function, and $\Phi_P$ denotes the multivariate standard normal distribution function with correlation matrix $P$
Gaussian copula applied in generating synthetic population

• Perform a cholesky decomposition of correlation matrix $P$, and set $A$ as the resulting lower triangular matrix

• Repeat the following steps $n$ times
  • Generate a vector $Z = (Z_1, \ldots, Z_d)'$ of independent standard normal deviates
  • Set $X = AZ$
  • Return $U = (\Phi(X_1), \ldots, \Phi(X_d))'$

• Can be achieved using `mvrnorm` in MASS library
R packages introduction

• A Grammar of Data Manipulation (dplyr)
  • R-package used for data processing
  • Transform and summarize tabular data with rows and columns.
  • Contain a set of functions (or “verbs”) that perform common data manipulation operations

• Create Elegant Data Visualizations Using the Grammar of Graphics (ggplot2)
  • R-package used for data visualization
  • Consistent underlying grammar of graphics (a graphic version of dplyr)
  • Plot specification at a high level of abstraction
  • Very flexible and elegant
R packages introduction - dplyr

• Commonly used command:
  • **mutate()** adds new variables that are functions of existing variables
  • **select()** picks variables based on their names
  • **filter()** picks cases based on their values
  • **summarise()** reduces multiple values down to a single summary
  • **arrange()** changes the ordering of the rows
  • **group_by()** allows for group operations in the “split-apply-combine” concept
R packages Introduction – dplyr - Continue

• Base R:
  • Create new dataset:
    
    ```r
    pop_census$new1 <- pop_census$x10 + 0.5 * pop_census$x11
    pop_census$new2 <- pop_census$x13 + 3 * pop_census$x14
    pop_census$new3 <- log(pop_census$x15)
    pop_census$new4 <- pop_census$x16 * 4
    ```
  • Select Variable:
    
    ```r
    pop_census[, c('X14', 'X15', 'X16', 'new1', 'X18')]
    ```
  • Filter variable:
    
    ```r
    pop_census [pop_census$x10 >= 3 & pop_census$x16 >= 1 & pop_census$x17 <= 7,]
    ```
  • Sort by variable value:
    
    ```r
    pop_census [ order(pop_census$x19),]
    ```
R packages Introduction – dplyr - Continue

• dplyr
  • This can be done with one function using dplyr:

```r
pop_census <- pop_census %>%
  mutate(new1 = X10 + 0.5 * X11,
         new2 = X13 + 3 * X14,
         new3 = log(X15),
         new4 = X16 * 4)

pop_census %>%
  dplyr::select(X14, X15, X16, new1, X18)
  %>%
  filter(X10 >= 1, X16 >= 1, X17 <= 7)
  %>%
  arrange(X19)
  %>%
  group_by(x2)
  %>%
  summarise(avg_X15 = mean(X15),
            min_X15 = min(X15),
            max_X15 = max(X15),
            total = n())
```
R packages introduction – ggplot2

• Graphic version of dplyr (using ‘+’ to replace ‘%>%’)

• Building blocks of a graph include:
  • data
  • aesthetic mapping
  • geometric object
  • statistical transformations
  • scales
  • coordinate system
  • position adjustments
  • faceting
• base R

```r
plot(x12 ~ x13,
     col = factor(x2),
     data = filter(pop_census, x2 %in% c("levels", "levelv")))
legend("topleft",
       legend = c("levels", "levelv"),
       col = c("red", "black"),
       pch = 1)
```
R packages Introduction – dplyr - Continue

• ggplot2

```r
ggplot(filter(pop_census, x2 %in% c("levels", "levelv")), 
aes(x=x13, 
y=x12, 
color=x2))+
geom_point()
```
R packages Introduction – ggplot2 - Continue

• More advanced features

```r
pop_census$pred.X12 <- predict(lm(X12 ~ X13, data = pop_census))

p1 <- ggplot(pop_census, aes(x = X13, y = X12)) + theme(legend.position="top",
                                                  axis.text = element_text(size = 6))

p1 + geom_point(aes(color = X12), alpha = 0.5,
                size = 1.5,
                position = position_jitter(width = 0.25, height = 0)) +
geom_line(aes(y = pred.X12)) + geom_smooth()
```
Step by step R function

• Categorical variables are converted to continuous before using copula
  • The frequency of the categorical variables’ levels are used as the value for that level

```r
dat[, cat] = apply(dat[, cat], 2, function(x){
  t = as.data.frame(table(x))
  t$Freq[match(x, t[, 1])]/length(x)
})
```

• CDF of the variable are transformed to be used in copula
  • $U_i$ is transformed to $\Phi_i^{-1}(u_i)$ using `ecdf` and `qnorm`

```r
prepare_copula_qnorm <- function (var){
  #This function is designed to calculate the copula
  new_var = qnorm(ecdf(var)(var)*0.99 + 0.005)
  return (new_var)
}
```
Step by step R function - Continue

dat <- dat %>% mutate (New_X1 = case_when( X1 == 1 ~ 1,
                   X1 ==1 ~ 2,
                   TRUE~ 0 ),
New_X2 = case_when((X2>=9 & X2 <=16) ~ 2,
                   TRUE~ 1),
New_X3= case_when(X3<=9 ~ 1,
                   (X3>=10 & X3 <=49) ~ 2,
                   (X3>=50 & X3 <=69) ~ 3,
                   (X3>=70 & X3 <=99) ~ 4,
                   (X3>=100 & X3 <=139) ~ 5,
                   (X3>=140 & X3 <=179) ~ 6,
                   (X3>=180 & X3 <=219) ~ 7,
                   (X3>=220 & X3 <=259) ~ 8,
                   (X3>=260 & X3 <=499) ~ 9,
                   (X3>=500 & X3 <=999) ~ 10,
                   (X3>=1000 & X3 <=1999) ~ 11,
                   (X3>=2000) ~ 12))

dat[,cat] = apply(dat[,cat],2,function(x){
    t = as.data.frame(table(x))
    t$Freq[match(x,t[,1])]/length(x)
})
```r
dat <- dat %>% mutate(
    New_X1_continuous = prepare_copula_qnorm(dat$New_X1),
    New_X2_continuous = prepare_copula_qnorm(dat$New_X2),
    New_X3_continuous = prepare_copula_qnorm(dat$New_X3),
    x4_continuous = prepare_copula_qnorm(dat$x4),
    x5_continuous = prepare_copula_qnorm(dat$x5),
    x6_continuous = prepare_copula_qnorm(dat$x6),
    x7_continuous = prepare_copula_qnorm(dat$x7),
    x8_continuous = prepare_copula_qnorm(dat$x8),
    x9_continuous = prepare_copula_qnorm(dat$x9),
    x10_continuous = prepare_copula_qnorm(dat$x10),
    x11_continuous = prepare_copula_qnorm(dat$x11),
    x12_continuous = prepare_copula_qnorm(dat$x12),
    x13_continuous = prepare_copula_qnorm(dat$x13),
    x14_continuous = prepare_copula_qnorm(dat$x14),
    x15_continuous = prepare_copula_qnorm(dat$x15),
    x16_continuous = prepare_copula_qnorm(dat$x16),
    x17_continuous = prepare_copula_qnorm(dat$x17),
    x18_continuous = prepare_copula_qnorm(dat$x18),
    x19_continuous = prepare_copula_qnorm(dat$x19),
    x20_continuous = prepare_copula_qnorm(dat$x20),
    x21_continuous = prepare_copula_qnorm(dat$x21),
    x22_continuous = prepare_copula_qnorm(dat$x22),
    x23_continuous = prepare_copula_qnorm(dat$x23),
    x24_continuous = prepare_copula_qnorm(dat$x24),
    x25_continuous = prepare_copula_qnorm(dat$x25)
)
return (dat)
}
```
create_newpop sizetype <- function(m, n, pop_num, dat_tol) {
  #This function is designed to create synthetic population within each New_X2 and New_X3
  dat_continuous <- dat_tol %>%
    filter(New_X2 == m, New_X3 == n) %>%
    dplyr::select(New_X1_continuous, X4_continuous, X5_continuous,
                  x6_continuous, x7_continuous, x8_continuous, x9_continuous, x10_continuous,
                  x11_continuous, x12_continuous, x13_continuous, x14_continuous, x15_continuous,
                  x16_continuous, x17_continuous, x18_continuous, x19_continuous, x20_continuous,
                  x21_continuous, x22_continuous, x23_continuous, x24_continuous, x25_continuous)
  dat_cor <- cor(dat_continuous)
  #generating new population
  new_pop <- mvnrnorm(n = pop_num, mu = rep(0, nrow(dat_cor)), dat_cor, tol = 1e-6, empirical = FALSE, EISPACK = FALSE)
  return (new_population)
}
Pairwise correlation comparison
Summary

• This study
  • Generating synthetic population by Gaussian copula
  • Adopting two R packages dplyr and ggplot2 simplified the tasks for this study

• dplyr and ggplot2 are two useful R packages
  • More convenient to manage
  • Easy to read
  • Decrease the workload
  • My personal preference to use over base R
References


Any Questions?

Thank you!